# Inquiry of Relationship between Ratio \& Proportionality Terms with Cognitive and Meta-Cognitive Orientation among the Preparatory Level Students 

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#### Abstract

The subject of the research was selected by the researcher's experience in mathematics teaching and by the fact that research continued with many mathematics teachers in finding that students had to be aware of proportion and proportionality. The researcher worked on a form showing the extent to which students in junior high were able to recognize the definitions of ratio and proportionality terms and their relationship to cognitive and abovecognitive attitudes in the proportional thinking of elementary level students. (Torre, 2013) indicated that most students must graduate from school knowing relative thinking. Therefore, students must be able to distinguish between proportional and non-proportional attitudes and use the appropriate method for each situation. (Misnasanti, Suwanto, 2017) noticed that the existence of gaps nowadays in identifying those elements directly is related to the ability to use ratios. Thus, applying relative logic is more complex than often thought and the way the concept of conceived relative thinking is more complicated. In this research, I will expand the idea through a form that answers the following question: What are the advantages of definitions of ratio and probability terms that are appropriate and what is their relationship to cognitive and above-cognitive trends in student proportional thinking? The results were there was moderate degree among all students in the proficiency of definitions of ratio and proportionality terms and their relation to cognitive and abovecognitive orientation in proportional thinking in students at the preparatory level. As for verbal math questions and proportional representation of fracture, ninth grade students have the ability to solve those questions more than others do. Finding the missing number requires higher skill ability, which was beyond seventh and eighth grade levels, so we found that ninth grad students had the ability to solve problems. With regard to meta-cognitive exercises, ninth grade students had the ability to master them more than others because these kinds of questions need to be worked out by students with some effort.


## Keywords:

Ratio, proportionality, cognitive orientation, metacognitive orientation, proportional thinking.

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## 1. INTRODUCTION

The problem of the study is revealed through the gaps nowadays that identify those elements in addition to the ability to use ratios and apply relative logic. This sounds more complicated than often thought because of the way in which the concept of relative thinking is indirectly implied by the tasks that have been included in relevant research and in mathematical textbooks. We can assume that relative logic is traditionally synonymous with the ability to solve the appropriate lost value (Misnasanti, Suwanto, 2017). This ability is the key element that justifies the inclusion of an aspect of analog thinking in the interpretation of relative logic, from very early on the existence of a relationship between ratios and measurements by indicating that the ratio is a special case of measurement. In fact, both ratios and measurements require students to think about relationships (Refaie, 2017).

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46

The subject of ratios and proportionality is an important central area in mathematics education and learning. The aim is to give students the concepts, generalizations, skills and means of thinking about the subject of proportion and proportionality, and to address and apply them in all other courses, whether on mathematics or on physics, geography, technology, etc., which makes it necessary for daily life. The proportions were widely used to solve the problems of everyday life (Refaie, 2017). Therefore, in mathematics and science, if relative logic is not understood conceptually and algorithmically, it becomes difficult to transmit and use it in everyday life. Therefore, relative thinking plays an important role in student mathematical development because it forms the backbone of mathematics curricula and includes important and interrelated ideas such as multiplication, division, fractions, decimal fractions, ratios, percentages and linear functions. It takes time, a variety of situations and opportunities to build their understanding in multiple ways. (Classroom Educator, 2012 (

The ratio (Atabas, 2013) is a comparison of two quantities while the rate is a comparison of two quantities with the opposite measures. For example of this is a ratio of 3 girls to 4 boys, and a rate of 4 miles in 5 hours. Even with the distinction between ratio and rate, there is no agreement to call the same idea a common term.

Proportionality is a description between two quantities. These are problem solving and calculation activities in areas that include scope, probability, percent, rate, trigonometry, equivalence, measurement, and flying shape engineering (Misnasanti, Suwanto, 2017).

Cognitive orientation is a set of higher skills that manage thought activities when an individual is involved in a situation of solving a problem or making a decision (Kahil, 2015).

Proportional thinking ( $\mathrm{J}, 2013$ ) means "a form of mathematical thinking that is related to the concepts of ratio and proportionality (the notion of ratio and proportionality, peripheral proportionality and inverse proportionality, proportional division) and their applications in mathematics, other sciences and working life."

Research was chosen to examine the relationships between a definition of ratio terms and their proportionality and their relationship to cognitive orientations in student proportional thinking. Student assistance is important because students consider it very difficult, and it is the basis for a wide range of mathematical concepts. Students find it difficult to understand ratios and proportionality because of their previous lack of understanding that extends to later years and the ability to recognize multiple comparisons and relationships between quantities, so there are difficulties for students in this subject.

In order to achieve the research objectives, a test is held to recognize the strategies of how students deal with problem relativity. The test consists of a set of questions and was intended to obtain information about the students' previous knowledge, and their strategies for solving the relative problem (Lukito, Nasument, 2015).

A set of hypotheses has been formulated:
The concept of proportion and proportionality.
Strategies for metacognition issues defined in proportion.

- Comparison of ratios.
- Finding the missing dimension in verbal matters.
- Determining proportionality.
- Proportional representation as fractions.
- Finding the missing dimension.
- Recognition of the notion of ratio in verbal matters.
- Metacognition proportion.


# Journal of Positive School Psychology 

2022, Vol. 6, Issue 2
Pp 22-46

## 2. THEORETICAL BACKGROUND

## 1) The concept of ratio and proportionality

The concept of ratio and proportionality is the relationship between two measured amounts, which are said to be proportional if the change is related to the change of one another by a fixed ratio. Two discounted amounts are proportional when the increase of one by a fixed ratio or number is related to the increase of the other by a fixed ratio or number. The two amounts are inversely proportional when the increase is related to the decrease of the other by a fixed ratio or number. The ratio being the comparison of two numbers or a fixed number. (Misnasanti, Suwanto, 2017).

## 2) Concept and types of thinking

Thinking is a set of mental processes that represent the imagination, the imaginary image, the understanding, the reflection on ideas and decision-making. It is a series of invisible and intangible mental activities that are relevant to the brain, in which the brain, when exposed to the excitement of what is received from one of the senses, or more of the sensations. The result of thinking represents the sum of the forms and processes of mind performed by the mind. (Refaie, 2017).

Mathematical thinking is a mental activity in mathematics. It comprises the following nine methods: extrapolation, extrapolation, generalization, formality, mathematical proof, symbol expression, visual perception, relational thinking, probabilistic thinking, when confronted with a problematic situation for which a solution is sought (Cetin \& Ertekin, 2011).

Mathematical thinking also involves different types of thinking, which vary depending on the mathematical theme .There is algebraic, probabilistic, statistical, geometric, proportional and many others. These types may overlap; requiring learners to be able to practice mathematical thinking of different kinds, especially since the interdependence of its subjects, the dependence of each on others, and the relevance of many of its subjects characterize mathematics to the reality of living life (Makdadi, 2017).

Proportional thinking is a way of thinking. It involves feeling and sense of quantitative relationships, comparison of proportions and fragmentation of information stored and needed for each context. It also represents the ability to understand, translate and solve issues associated with different proportional situations using beat, and relative and comparative thinking (classroom Educator, 2012).

A study of Makdadi (2017) showed a sample of 523 Female students from the scientific and literary secondary schools of the State Schools of Education and Education Departments of Luwayi Al-Tawayyah and Al-Mu'ayyah and Bani Abid Brigade, if at all. Of the female students, $25.05 \%$ were classified at first proportional levels of thinking. (Very low) ( $60.04 \%$ ) Female students rated at first proportional levels of thinking (Weak), ( $14.53 \%$ ) Female students rated at First Proportional Thinking Levels (average), ( $0.38 \%$ ) of female students were classified in the first (empowered) levels of proportional thinking. The level of proportional thinking of female students is concentrated in the second (weak) level.

Jarradat (2013) conducted a study of the first secondary students in Saudi Arabia to uncover the relationship between the level of proportional thinking and the level of probabilistic thinking of the students. The results revealed a statistically significant relationship between the ability to think proportional and the ability to think probabilistic to students.

## 3) Proportional thinking components:

1. Relative thinking:

It's based on understanding the relationships between the different factors in the situations or problem

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46
facing the individual and the mathematical issue that has a number of elements. if the pupil is properly aware of the relationship between them, it leads him to the right solution (Refaie, 2017).
2. Units and unitizing:

It represents the concepts and representation of numbers and numbers systems (including conversion
between numbers), including the properties of integer numbers and relative numbers, and the relevant aspects of non-proportional numbers, as well as quantities and units that refer to phenomena such as time and money, weight, temperature, distance, area, size and derivative quantities and their numerical description (Mclntosh, 2013).

## 3. Partitioning:

This part of represents the student's process of deconstructing the parts so that these parts together represent the whole. This is shown by the combination of fractures so that all fractures represent an integer number. The division lies at the heart of the understanding of the relative number, as students divide a whole number into a number of equal parts (Refaie, 2017).
4. Attending to quantities and change:

Quantity is the most widespread and important mathematical aspect so that we can participate in and interact with our world. This concept includes quantifying the characteristics of objects, relationships, attitudes and entities in the world, understanding the different representations of such quantities, and judging quantitative interpretations and proofs. The link to the quantification of the world includes an understanding of benchmarks, numbers, amounts, units, indicators, relative size, trends and digital patterns.

## 5. Ratio sense:

Students skilled in mathematics are able to make sense of proportion and unit rates in the context of realworld attitudes, and persevere when choosing and using appropriate representations of given contexts. The development of a percentage of students requires attention to fractional representations as a means of arranging and equalizing ratios in the context of content that illustrates the use of ratios. There are four types of ratios that are important in solving proportional problems: Part, Whole part, associated groups, and wellknown metrics. Students always need to make sure that they describe relationships more than just processing sets of numbers (Refaie, 2017).

## 6. Rational numbers interpretations:

Relative numbers are numbers that are found in the form of a two-digit ratio. The relative number is a fraction consisting of a commutation. To illustrate the comparison between the relative numbers and how the relative numbers are arranged by explaining the comparative lesson of the relative numbers and their order, the student must know the mechanism of dealing with the relative numbers correctly, where many arithmetic processes can be done on the relative numbers (Ortiz 2015).

## 4) Levels of proportional thinking:

(Ortiz, 2015) states that there are four levels of proportional thinking:
a) Below level one: This level depends on guesses and the use of sight to estimate certain things. This level of thinking is so disproportionate that the connection between the two amounts is not accurate.
b) Level one: This level is more comprehensive than the above. It depends on the dimension of the images and the strategies through which the sense of situations that are related to comparisons is relied upon. This level of thinking is indicative but not formal in proportional situations.

# Journal of Positive School Psychology 

2022, Vol. 6, Issue 2
Pp 22-46
c) Level two: Inference depends on many skills from equidistant breakdowns, metrics and strategies that express the relative aspect. At this level the inference is quantum.
d) Level 3: At this level, the arithmetic operations of a multiplication and division are used in order to reach the appropriate proportionality that expresses the relations which bind equal fractions.

Teacher's role in teaching proportion and proportionality:
To help students develop their thinking, they need educational opportunities and expertise to develop their conceptual knowledge of proportionality. So, they are able to distinguish situations that require proportional reflection. The role of the teacher in the development of a series of tasks is highlighted during the curriculum for the development of student capacity. There is a set of requirements for the teacher to perform while teaching students the subject of proportion and proportionality. For example, linking the difference between fracture units and rate units, students are strengthened in order to increase their motivation to extract proportions and create proportional relationships (Hitton et al., 2013).

## 5) Cognitive Trends in Proportional Thinking

Proportional thinking is a concept and skill that plays a role in many of the mathematical subjects studied by the student. It is not just a conserved procedure or algorithm that is applied, it includes a conceptual understanding of proportional relationships. Proportional thinking requires a genuine understanding of the concepts of ratio and proportionality, and requires the ability to use concepts appropriately to solve and evaluate different situations of issues. It needs a higher thinking skills and the ability to distinguish between summation and multiplication comparisons. Proportional thinking also distinguishes between peripheral and reverse proportionality. With proportional thinking, students need to develop strategies, do high-level cognitive tasks to have the abilities beyond the perceived cognition to observe and judge the reliability of their thinking without direct instructions. (Putarek, 2019).

## Research method

Research question: What are the advantages of ratio definitions and what are their relation to cognitive metacognitive trends in student proportional thinking?

## Research method

Quantitative method is used in this research. Ratio is tracked and adapted to the cognitive and metacognitive orientation of students through forms that are distributed to students at the junior high level. Results recognize the stages of relative thinking in a clear and understandable way.

The research form consisted of an article examining ratio terms and their relevance to cognitive and metacognitive orientations. It discussed levels of proportional thinking. The form has 9 questions in ratio which cover all levels of thinking.

The form is modified to suit the students. The form tested the relevance and motivation of each student.

| 1 | Define the concept of ratio and proportionality | Cognitive examination |
| :--- | :--- | :--- |
| 2 | A 9-year-old's boy is 1.23 meters tall. How tall would the <br> boy be at the age of $18 ?$ | Metacognitive exercise |

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46

| 3 | In eighth grade 15 boys and 10 girls <br> * What is the ratio between boys and girls? <br> *What is the ratio between the number of boys and the number of all class students? | Comparison of ratios |
| :---: | :---: | :---: |
| 4 | In a dairy factory, you need 10 liters of milk to produce 2 kg of butter. How many liters of milk you need to produce 6 kg of butter? | Do verbal math exercise |
| 5 | In a building, the fire system controls 9 out of 10 fires. How many fires can the system control if there are 20 fire? <br> A: 9 B:10 C:18 D:20 | Comparative exercises |
| 6 | Write (T) in front of the correct proportion: $2 / 5,4 / 5$ $1 / 3,3 / 6$ $3: 2,9: 6$ | proportional representation of fraction |
| 7 | $(2,4,16)$ ------ $(6: 3 \cdot 8)$ <br> $(1,3,6)$ ------ $(6: 4 \cdot 9)$ <br> $(72,64,37)$ ------ $(6: 36 \cdot 8)$ | Identifying numerical similarity |
| 8 | A chocolate cake for 3persons needs 120 grams of chocolate, 9 large spoons of cream, 3 eggs, 4 large spoons of coffee, 4 large spoons of sugar. If we want to make a cake for four people instead of three. How much sugar do we need | Identification of the notion of ratio in verbal questions. |
| 9 | To paint a 4 cm square image, we need an 8 mm in real one. How much color do we need to paint the magnifying image into a square of a 12 cm of each side? | Metacognitive exercise |

## 3. RESEARCH POPULATION AND SAMPLES

- Research population: preparatory school in the South that was founded in 1944.
- Research samples: The sample consisted of 55 students from lower secondary schools in the Arab region. The form was applied to them in the subject of mathematics during the 2021 school year. There is a body of feminist literary criticism of plays. Some of this is first stage "image of women" criticism, which points out patterns in writing by men in which, for example, the female characters suffer or die in or- die for the male characters to grow or continue on their life journeys. Other, second-stage criticism focuses on patterns in women's writing such as repressed fears and anger expressed through coded plot lines and character types. Third-stage theory-centered work tends to focus on language itself and the connections or lack thereof between words and reality (21).


## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46

## 4. RESEARCH PROCEDURE

The form was distributed in 2021 to a preparatory school. 55 students from seventh to ninth grades participated in the. The form included 9 questions, which were presented to them as a test. There was enough time for the students to solve the test according to the age group.

## 5. RESULTS

1. Do cognitive and meta-cognitive attitudes in proportional thinking of preparatory students influence the notion of proportion and proportionality?

To answer the previous question, the response ratios were calculated to the question that: Define the concept of proportion and proportionality?

|  | Seventh grade |  |
| :--- | :--- | :--- |
| Percentage of the correct answers |  | $35.7 \%$ |
| Arithmetic Averages |  | 0.714 |
|  | Eighth grade |  |
| Percentage of the correct answers |  | $45.0 \%$ |
| Arithmetic Averages |  | .900 |
|  | Ninth grade |  |
| Percentage of the correct answers |  | $40.9 \%$ |
| Arithmetic Averages |  | 0.817 |

The percentage of the seventh grade answer was $35.7 \%$. The percentage of the eighth grade was $45.0 \%$. The ninth grade percent was $40.9 \%$. The answers of the students varied according to their age. Their perspective on the interpretation of the question varied because the majority of the students knew proportionality incorrectly.

The following is a graph of the answers percentage:


## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46
The following chart compares the answers of preparatory level students:


From the above graph, it is clear that the effect of cognitive and meta-cognitive attitudes in proportional thinking of preparatory students on the concept of ratio and proportionality in eighth grade was $37 \%$ the highest, while it was $34 \%$ for ninth grade and $29 \%$ for seventh grade.
2. Is there is an influence of cognitive and meta-cognitive attitudes in the proportional thinking of students on strategies for solving meta-cognitive issues in proportion?

To answer the previous question, the response ratios were calculated about the question: If the 9 -yearold boy is 1.23 meters tall, how tall will be at the age of 18 ?

|  | Seventh grade |  |
| :---: | :---: | :---: |
| Percentage of the correct <br> answers |  | $89.3 \%$ |
| Arithmetic Averages | Eighth grade | 1.78 |
|  |  | $65.0 \%$ |
| Percentage of the correct <br> answers | Ninth grade | 1.30 |
| Arithmetic Averages |  | $72.6 \%$ |
| Percentage of the correct <br> answers |  | 1.45 |
| Arithmetic Averages |  |  |

The percentage of students who answered correctly in seventh grade was $89.3 \%$. The percentage of eighth grade students were $65.0 \%$ and ninth grade was $72.6 \%$.

The following graph illustrates each class by the correct answers.

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46


The following graph compares the student's answers in the three levels:


The above graph clearly indicates that the influence of cognitive and meta-cognitive attitudes in the proportional thinking of preparatory students of the seventh grade was $39 \%$. The ninth grade was $32 \%$ and the eighth grade was $29 \%$.
3. Is there is an influence of cognitive and meta-cognitive attitudes in proportional thinking of students on the comparison of ratios?

To answer the previous question, the ratios have been calculated for the following questions:

- What is the ratio of boys to girls?
- What is the ratio of boys to the whole class?

|  | Seventh grade |  |
| :---: | :---: | :---: |
| Percentage of the correct <br> answers |  | $96.4 \%$ |
| Arithmetic Averages |  | 1.92 |
|  | Eighth grade |  |

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46

| Percentage of the correct <br> answers |  | $67.5 \%$ |
| :---: | :--- | :---: |
| Arithmetic Averages |  | 1.35 |
|  | Ninth grade |  |
| Percentage of the correct <br> answers |  | $37.8 \%$ |
| Arithmetic Averages |  | 1.47 |

One explanation of the students' answer is misunderstanding of the question. They thought that they should answer according to the class they are in. Some students complained and said it was difficult to answer. The percentage of correct answers in seventh grade was $96.4 \%$, in ninth grade it was $73.8 \%$ and in eighth grade was $67.5 \%$.

The following graph indicates the answers of each class:


The following chart compares student's answers of preparatory level:


The above chart showed the effect of cognitive and meta-cognitive attitudes in relativistic thinking of preparatory students. The seventh grade ratio was $41 \%$, which was the highest, compared with ninth grade (31\%) and eighth grade (28\%).
4. Is there an influence of cognitive and meta-cognitive attitudes in the proportional thinking of students

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46
on finding the missing dimension in verbal matters?
To answer the previous question, the response ratios were calculated to the following question:
In a dairy factory, you need 10 liters of milk to produce 2 kg of butter, so how many liters of milk do you need to produce 6 kg of butter?

|  | Seventh grade |  |
| :---: | :---: | :---: |
| Percentage of the correct <br> answers |  | $60.7 \%$ |
| Arithmetic Averages | Eighth grade | 1.21 |
| Percentage of the correct <br> answers |  | $72.5 \%$ |
| Arithmetic Averages | Ninth grade | 1.45 |
| Percentage of the correct <br> answers |  | $80.0 \%$ |
| Arithmetic Averages |  | 1.59 |

The answers showed that the 7th grade had $60.7 \%$ correct answer. The 8 th grade had $72.5 \%$ correct answer and the 9th grade had $80 \%$ correct answers.

The following graph shows the correct answers of each class:


The following chart compares the student's answers of each level:

# Journal of Positive School Psychology 

2022, Vol. 6, Issue 2
Pp 22-46


From the above chart, we notice that the effect of cognitive and meta-cognitive trends in proportional thinking of preparatory level students of finding the missing dimension was $38 \%$ for the ninth grade, while it was $34 \%$ in eighth grade and $28 \%$ in the seventh grade.
5. Do cognitive and meta-cognitive attitudes in the proportional thinking of students influence the determination of proportionality?

To answer the previous question, the response ratios were calculated to the following question: The fire control system in a house controls nine out of every 10 fires. How many fires can be controlled from the same house of if there are 20 fires?

|  | Seventh grade |  |
| :--- | :--- | :--- |
| Percentage of the correct <br> answers |  | $100 \%$ |
| Arithmetic Averages | Eighth grade | 2.00 |
|  |  | 72.5 |
| Percentage of the correct <br> answers | Ninth grade | 1.45 |
| Arithmetic Averages |  | 82.4 |
|  |  | 1.64 |
| Percentage of the correct <br> answers |  |  |
| Arithmetic Averages |  |  |

Explanations have been given to students with similar examples. This kind of question, which is aimed at determining proportionality, has monitored students' potential in this kind of question. The seventh class percentage of correct answer was $100 \%$, the ninth was $82.4 \%$, and the eighth grade was $72.5 \%$.

The following graph illustrates the correct answers of each class:

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46


The following chart compares the student's answers of each level:


We note from the chart that the effect of cognitive and meta-cognitive attitudes in the proportional thinking of students in the seventh grade was $39 \%$, which is the highest. The next was the ninth grade that was $32 \%$, and finally is the eighth grade, $29 \%$.
6. Do cognitive and meta-cognitive attitudes of proportional thinking have an influence on fractions concept of the preparatory level students?

To answer the above question, students' answers were calculated for the following question:
Write (T) in front of the correct answer:
$2 / 5,4 / 5 \quad(\quad)$
$1 / 3,3 / 6 \quad(\quad)$
3:2, 9:6 ( )

|  | Seventh grade |  |
| :---: | :---: | :---: |
| Percentage of the correct <br> answers |  | $23.8 \%$ |
| Arithmetic Averages |  | 0.71 |
|  | Eighth grade |  |
| Percentage of the correct |  | $26.7 \%$ |

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46

| answers |  |  |
| :---: | :---: | :---: |
| Arithmetic Averages |  | 0.8 |
|  | Ninth grade |  |
| Percentage of the correct <br> answers |  | $38.1 \%$ |
| Arithmetic Averages |  | 1.14 |

It is clear that seventh grade had the highest correct answers. The percent was $71 \%$. The ninth grade percent was $38.1 \%$, and the percentage of correct answers for class 9 was $26.7 \%$. That is, the student proficiency to solve questions about fractional representation was problematic.

The following graph illustrates the correct answers of each class:


The following chart compares the student's answers of each level:


We note from the chart that the effect of cognitive and meta-cognitive attitudes in proportional thinking of preparatory level students on proportional representation of fractions in ninth grade was $43 \%$, while $30 \%$ for eighth grade and $27 \%$ for seventh grade.
7. Do cognitive and meta-cognitive attitudes of proportional thinking have an influence on students concept when answering finding the missing dimension?

To examine the previous question, we calculated the answers of the following question:
$6: 3 \cdot 8$ $\qquad$ $(2,4,16)$
6:4•9 $\qquad$ $(1,3,6)$

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46
6:36•8 $\qquad$ (37, 64, 72)

|  | Seventh grade |  |
| :---: | :---: | :---: |
| Percentage of the correct <br> answers |  | $28.6 \%$ |
| Arithmetic Averages | Eighth grade | 0.85 |
| Percentage of the correct <br> answers |  | $26.7 \%$ |
| Arithmetic Averages |  | 0.80 |
| Percentage of the correct <br> answers |  | $35.8 \%$ |
| Arithmetic Averages |  | 1.07 |

The results as shown above: Ninth grade was $35.8 \%$, seventh grade was $28.6 \%$ and eighth grade was $26.7 \%$.

The following graph illustrates the correct answers of each class:


The following chart compares the students answers of each level:

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46


We note from the chart that the effect of cognitive and meta-cognitive attitudes in proportional thinking of students on finding the missing dimension in ninth grade was $39 \%$ that is more than the seventh grade $32 \%$. Eighth grade percent is $29 \%$.
8. Do cognitive and meta-cognitive attitudes of the proportional thinking of elementary level students affect the perception of ratio in verbal math questions?
to answer the above question, The ratios response had been calculated to the following question:
a chocolate cake needs 120 grams of chocolate, 9 large spoons of cream, 3 eggs, 4 large spoons of coffee, and 4 large spoons of sugar for three people. If we want to make a cake for four people instead of three. How much sugar do we need?

|  | Seventh grade |  |
| :---: | :---: | :---: |
| Percentage of the correct <br> answers |  | $78.6 \%$ |
| Arithmetic Averages | Eighth grade | 1.57 |
| Percentage of the correct <br> answers |  | $60.0 \%$ |
| Arithmetic Averages | Ninth grade | 1.20 |
| Percentage of the correct <br> answers |  | $66.4 \%$ |
| Arithmetic Averages |  | 1.32 |

For some students, the proficiency of this type of question in the 7th grade was $78.6 \%$ correct, 9 th grade was $66.4 \%$ and 8th grade was $60 \%$.

The following graph illustrates the correct answers of each class:

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46


The following chart compares the students answers of each level:


We note from the chart above that the effect of cognitive and meta-cognitive attitudes of proportional thinking of preparatory level students on the perception of ratio in verbal math questions of seventh grade was $38 \%$ higher, while $33 \%$ was for ninth grade, and $29 \%$ was for eighth grade.
9. Do cognitive and meta-cognitive attitudes of the proportional thinking of elementary students affect meta-cognitive issues in proportion?

To answer the previous question, the ratios responses were calculated to answer the following question: 8 mm is required to paint a 4 cm square image. How much paint do we need to paint the magnified image of a 12 cm square?

|  | Seventh grade |  |
| :---: | :---: | :---: |
| Percentage of the correct <br> answers |  | $50.0 \%$ |
| Arithmetic Averages | Eighth grade | 1.0 |
| Percentage of the correct <br> answers |  | $65.0 \%$ |
| Arithmetic Averages | Ninth grade | 1.30 |

# Journal of Positive School Psychology 

2022, Vol. 6, Issue 2
Pp 22-46

| Percentage of the correct <br> answers |  | $79.7 \%$ |
| :---: | :--- | :--- |
| Arithmetic Averages |  | 1.59 |

The percentage of student's answers of this question of the ninth was $79.7 \%$. The eighth grade answers percent was $65.0 \%$. The seventh grade answers percent was $50.0 \%$.

The following graph illustrates the correct answers of each class:


The following chart compares the students answers of each level:


We note from the chart that the effect of cognitive and meta-cognitive attitudes of the proportional thinking of students on meta-cognitive issues of the ninth grade was $41 \%$, which is higher than eighth grade $33 \%$ and seventh grade $26 \%$.

## 6. DISCUSSION

Results of this study indicates that students have good understanding of the concept of proportion and proportionality. There is a convergence of students in the proficiency of that concept, because this subject is taught at various stages at school. Students of 7th, 8th, and 9th grades study the subject of proportion and proportionality and have a moderate understanding of those concepts. This is due to difference in environmental factors and intelligence level of students as well.

The strategies of math problem solving vary from student to another. For example, seventh grade students have the capacity to master these strategies more than other students. This is because seventh grade

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46
students have the most motivation towards learning. They can also be controlled and directed towards the strategies used to solve these problems than eighth and ninth graders because they have gone through a higher level of learning.

As comparison of ratios is concerned, seventh grade students have the ability to understand this concept because the lesson is done in a simplified way compared to ninth and eighth grade students.

Regarding the verbal math questions and the proportional representation of fracture, ninth grade students were able to solve the questions more than others. This is because they were able to read and understand the meaning of the questions. On the other hand, concerning the subject of determining descent, seventh grade students have the ability to master it more than other students.

Finding the missing number requires a higher skill capacity that was difficult for grade 7 and grade 8th to answer and easy for 9 th grade student. It also revealed that the concept of ratio was more advanced in grade 7 th students than others.

With regard to meta-cognitive math exercises, ninth grade students had the ability to master them more than others because these kinds of exercises need to be worked out by students.

Finally, there was a moderate degree for all students of the proficiency of definitions of ratio and proportionality terms and their relation to cognitive and meta-cognitive orientation in the proportional thinking of preparatory level students. The percentage ranged between $55.4 \%$ to $63 \%$ of all students.

Makdadi study (2017) showed that $25.05 \%$ of female students were rated for the first proportional thinking as (Very Low), $60.04 \%$ were rated as (Weak), $14.53 \%$ were rated as (Intermediate) and $0.38 \%$ were rated as (very good).

It is clear that the level of proportional thinking of female students is concentrated in the second level (weak), while a Garadat study (2013) shows a statistically relevant relationship between the ability proportional thinking and the probability thinking.

There is an intermediate degree among all students in the proficiency of definitions of ratio and proportionality terms and their relation to cognitive and meta-cognitive orientation in proportional thinking of students at the preparatory level.

The strategies for resolving math problems vary among students. Seventh grade students had the ability to master those strategies and the concept of ratio.

As for verbal questions and proportional representation of fracture, ninth grade students were able to solve the questions more than others.

Finding missing number and meta-cognitive questions also requires higher skill ability, which was beyond the seventh and eighth grade ability. This is explaining why ninth grade students succeeded in solving the questions easily.

Regarding meta-cognitive exercises, ninth grade students had the ability to master them more than others because these kinds of questions needed more efforts from students.

## 7. RECOMMENDATIONS

From the results, I recommend the following:

- To pay more attention to the development of proportional thinking and to make more research about the levels and development of proportional thinking in different categories of students.
- Teachers should provide an appropriate classroom environment that allows students to exchange views and encourages them to have the right answers by understanding of


# Journal of Positive School Psychology 

2022, Vol. 6, Issue 2
Pp 22-46
proportionality subject. Answers should be discussed logically.

- To develop students' levels of proportional thinking and suggest special training programs that contributes to the development of those levels.


## Appendix

## A. Math test on ratios and proportions:

1. Define the concept of proportion and proportion?

Answer: $\qquad$
2. If a 9 -year-old is 1.23 meters tall, his 18 th birthday will be his height?

Answer: $\qquad$
3. In the eighth grade, 15 boys and 10 girls:
i. What is the ratio between the numbers of boys to the number of girls?

Answer:
ii. What is the ratio between the numbers of children to the number of all students in the class?

Answer:
4. In a dairy factory, it takes 10 liters of milk to produce 2 kg of butter

How many liters of milk itself would you need to produce 6 kg of butter?
Answer:
5. The fire control system in a building controls 9 out of 10 fires, the number of fires that can be controlled from the house of 20 fires in the same system is:
a) 9
b) 10
c) 18
d) 20
6. Put a circle in front of the two ratios that represent the proportion
a) $2 / 5,4 / 5$
b) $1 / 3,3 / 6$
c) $9: 6,3: 2$
7. Calculate the following:
a) $6: 3 \times 8$
b) $6: 4 \times 9$
c) $6: 36 \times 8$
8. Description of Chocolate Cake for Three Persons 120 grams of chocolate, 9 tablespoons of cream, 3 eggs, 4 tablespoons of coffee, 4 tablespoons of sugar. If we wanted to make a cake for four people instead of three. How much sugar do we need?
Answer:
9. 8 mm is required to paint a 4 cm square picture, how much color do we need to paint the enlarged image into a square with a side of 12 cm ?
Answer:

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46
B. Statistical analysis:
i. Right versus wrong questions at every grade

|  | Seventh grade |  | Eighth grade |  | Ninth grade |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| question 1 |  |  |  |  |  |  |
|  | Correct answers | 35.7 | Correct answers | 45 | correct answers | 40.9 |
|  | Wrong answers | 64.3 | Wrong answers | 55 | wrong answers | 59.1 |
| question 2 |  |  |  |  |  |  |
|  | Correct answers | 89.3 | Correct answers | 65 | $\begin{gathered} \hline \begin{array}{c} \text { correct } \\ \text { answers } \end{array} \end{gathered}$ | 72.6 |
|  | Wrong answers | 10.7 | Wrong answers | 35 | wrong answers | 27.4 |
| question 3 |  |  |  |  |  |  |
|  | Correct answers | 96.4 | Correct answers | 67.5 | correct answers | 73.8 |
|  | Wrong answers | 3.6 | Wrong answers | 32.5 | wrong answers | 26.2 |
| question 4 |  |  |  |  |  |  |
|  | Correct answers | 60.7 | Correct answers | 72.5 | correct answers | 80 |
|  | Wrong answers | 39.3 | Wrong answers | 27.5 | $\begin{aligned} & \text { wrong } \\ & \text { answers } \end{aligned}$ | 20 |
| question 5 |  |  |  |  |  |  |
|  | Correct answers | 100 | Correct answers | 72.5 | correct answers | 82.4 |
|  | Wrong answers | 0 | Wrong answers | 27.5 | $\begin{gathered} \text { wrong } \\ \text { answers } \end{gathered}$ | 17.6 |
| question 6 |  |  |  |  |  |  |
|  | Correct answers | 23.8 | Correct answers | 26.7 | correct answers | 38.1 |
|  | Wrong answers | 76.2 | Wrong answers | 73.3 | wrong answers | 61.9 |
| question 7 |  |  |  |  |  |  |
|  | Correct answers | 28.6 | Correct answers | 26.7 | correct answers | 35.8 |
|  | Wrong answers | 71.4 | Wrong answers | 73.3 | wrong answers | 64.2 |
| question 8 |  |  |  |  |  |  |
|  | Correct answers | 78.6 | Correct answers | 60 | correct answers | 66.4 |
|  | Wrong answers | 21.4 | Wrong answers | 40 | wrong answers | 33.6 |
| question 9 |  |  |  |  |  |  |
|  | Correct answers | 50 | Correct answers | 65 | correct answers | 79.7 |
|  | Wrong answers | 50 | Wrong answers | 35 | wrong answers | 20.3 |

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46
ii. Percentage of right versus wrong questions to all grade:

|  | q1 | q2 | q3 | q4 | q5 | q6 | q7 | q8 | q9 | Final mark |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| arithmet ic average | $\begin{array}{r} 0.7142 \\ \hline 86 \\ \hline \end{array}$ | $\begin{array}{r} 1.7857 \\ \hline 14 \\ \hline \end{array}$ | $\begin{array}{r} 1.9285 \\ 71 \\ \hline \end{array}$ | $\begin{array}{r} 1.2142 \\ 86 \\ \hline \end{array}$ | 2 | $\begin{array}{r} 0.7142 \\ 86 \\ \hline \end{array}$ | $\begin{array}{r} 0.8571 \\ 43 \\ \hline \end{array}$ | $\begin{array}{r} 1.5714 \\ \hline 29 \\ \hline \end{array}$ | 1 | $\begin{array}{r} 11.785 \\ 71 \\ \hline \end{array}$ | Sevent h grade |
| Ratio | 35.7\% | 89.3\% | 96.4\% | 60.7\% | 100.0\% | 23.8\% | 28.6\% | 78.6\% | 50.0\% | 58.9\% | 62.2\% |
| arithmet ic average | 0.9 | 1.3 | 1.35 | 1.45 | 1.45 | 0.8 | 0.8 | 1.2 | 1.3 | 10.55 | Eighth grade |
| Ratio | 45.0\% | 65.0\% | 67.5\% | 72.5\% | 72.5\% | 26.7\% | 26.7\% | 60.0\% | 65.0\% | 52.8\% | 55.4\% |
| arithmet ic average | $\begin{array}{r} 0.8178 \\ \quad 57 \\ \hline \end{array}$ | $\begin{array}{r} 1.4511 \\ \hline \end{array}$ | $\begin{array}{r} 1.4767 \\ \quad 86 \\ \hline \end{array}$ | $\begin{array}{r} 1.5994 \\ 05 \\ \hline \end{array}$ | $\begin{array}{r} 1.6470 \\ 24 \\ \hline \end{array}$ | $\begin{array}{r} 1.1444 \\ 44 \\ \hline \end{array}$ | $\begin{array}{r} 1.0730 \\ 16 \\ \hline \end{array}$ | $\begin{array}{r} 1.3285 \\ 71 \\ \hline \end{array}$ | $\begin{array}{r} 1.5940 \\ 48 \\ \hline \end{array}$ | $\begin{array}{r} 12.025 \\ 65 \\ \hline \end{array}$ | Ninth grade |

iii. The average and percentages in 7th grade:

|  | q1 | q2 | q3 | q4 | q5 | q6 | q7 | q8 | q9 | sum |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pupil 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 2 | 2 | 20 | 7th |
| Pupil 2 | 1 | 2 | 2 | 1 | 2 | 0 | 0 | 2 | 2 | 12 | 7th |
| Pupil 3 | 1 | 2 | 2 | 1 | 2 | 0 | 0 | 2 | 0 | 10 | 7th |
| Pupil 4 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 19 | 7th |
| Pupil 5 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 2 | 14 | 7th |
| Pupil 6 | 0 | 2 | 2 | 1 | 2 | 0 | 0 | 2 | 0 | 9 | 7th |
| Pupil 7 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 18 | 7th |
| Pupil 8 | 0 | 2 | 2 | 1 | 2 | 0 | 0 | 2 | 0 | 9 | 7th |
| Pupil 9 | 0 | 2 | 2 | 1 | 2 | 0 | 0 | 0 | 0 | 7 | 7th |
| Pupil 10 | 2 | 1 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 18 | 7th |
| Pupil 11 | 0 | 2 | 2 | 1 | 2 | 0 | 0 | 2 | 0 | 9 | 7th |
| Pupil 12 | 0 | 2 | 2 | 1 | 2 | 0 | 0 | 0 | 0 | 7 | 7th |
| Pupil 13 | 1 | 2 | 2 | 1 | 2 | 2 | 1 | 2 | 2 | 15 | 7th |
| Pupil 14 | 0 | 1 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 5 | 7th |
| Pupil 15 | 1 | 2 | 2 | 1 | 2 | 0 | 1 | 2 | 2 | 13 | 7th |
| Average | 0.71428 | 1.785714 | 1.928571 | 1.2142 | 62 | 0.714286 | 0.85714 | 1.571429 | 1 | 11.78571 | 7th |

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46
$\begin{array}{lcccccccc}\text { Ratio } & 35.7 \% & 89.3 \% & 96.4 \% & 60.7 \% & 100.0 \% & 23.8 \% & 28.6 \% & 78.6 \% \\ 50.0 \% & 58.9 \% & 62.2 \% & & & & & & \end{array}$
iv. The average and percentages in 8th grade

|  | q1 | q2 | q3 | q4 | q5 | q6 | q7 | q8 | q9 | sum |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pupil 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 2 | 2 | 20 | 8th |
| Pupil 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 19 | 8th |
| Pupil 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 18 | 8th |
| Pupil 4 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 16 | 8th |
| Pupil 5 | 0 | , | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 8 | 8th |
| Pupil 6 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 6 | 8th |
| Pupil 7 | 1 | 2 | 2 | 2 | 2 | 1 | 0 | 2 | 2 | 14 | 8th |
| Pupil 8 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 5 | 8th |
| Pupil 9 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 15 | 8th |
| Pupil 10 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 4 | 8th |
| Pupil 11 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 8th |
| Pupil 12 | 1 | 2 | 2 | 2 | 2 | 0 | 0 | 2 | 2 | 13 | 8th |
| Pupil 13 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 15 | 8th |
| Pupil 14 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 3 | 8th |
| Pupil 15 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 8th |
| Pupil 16 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 2 | 2 | 19 | 8th |
| Pupil 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8th |
| Pupil 18 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 16 | 8th |
| Pupil 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8th |
| Pupil 20 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 2 | 2 | 17 | 8th |
| Average | 0.9 | 1.3 | 1.35 | 1.45 | 1.45 | 0.8 | 0.8 | 1.2 | 1.3 | 10.55 | 8th |


| Ratio | $45.0 \%$ | $65.0 \%$ | $67.5 \%$ | $72.5 \%$ | $72.5 \%$ | $26.7 \%$ | $26.7 \%$ | $60.0 \%$ | $65.0 \%$ | $52.8 \%$ | $55.4 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46
v. The average and percentages in 9th grade:

|  | q 1 | q 2 | q 3 | q 4 | q 5 | q 6 | q 7 | q 8 | q 9 | sum |  |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pupil 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 16 | 9 th |
| Pupil 2 | 1 | 2 | 2 | 2 | 2 | 2 | 1 | 2 | 2 | 16 | 9 th |
| Pupil 3 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 15 | 9 th |
| Pupil 4 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 17 | 9 th |
| Pupil 5 | 1 | 2 | 2 | 2 | 2 | 2 | 1 | 2 | 2 | 16 | 9 th |
| Pupil 6 | 0 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 13 | 9 th |
| Pupil 7 | 0 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 2 | 11 | 9 th |
| Pupil 8 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 2 | 2 | 20 | 9 th |
| Pupil 9 | 0 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 2 | 11 | 9 th |
| Pupil 10 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 14 | 9 th |
| Pupil 11 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 9 th |
| Pupil 12 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 17 | 9 th |
| Pupil 13 | 0 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 13 | 9 th |
| Pupil 14 | 0 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 2 | 10 | 9 th |
| Pupil 15 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 2 | 2 | 20 | 9 |

## Journal of Positive School Psychology

2022, Vol. 6, Issue 2
Pp 22-46

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